TRANSLATION

ON THE PRINCIPLE OF CONTRADICTION IN ARISTOTLE [1]

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IN THE TREATISE NAMED ABOVE the author set himself the task of subjecting to a thorough critique Aristotle's presentation of the principle of contradiction, primarily in *Metaphysics*, Gamma. The necessity for a review of the principle of contradiction seems to offer itself directly in the wake of the enormous progress of symbolic logic as founded by G. Boole and powerfully developed through the work of De Morgan, Pierce, Schröder, Frege, Peano, B. Russell, etc. One cannot conceal the fact that, *compared with*

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[1] This article originally appeared under the title *Über den Satz des Widerspruchs bei Aristoteles* in *Bull. Intern. de l'Académie des Sciences de Cracovie*, Cl. d'histoire et de philosophie, 1910. The article is based on a longer study which appeared in Polish the same year: *O zasadzie sprzeczności u Arystotelesa* (Concerning the Principle of Contradiction in Aristotle). This latter study was the most important of his early writings and figured influentially in the logical-philosophical renaissance of early twentieth century Poland. Lukasiewicz evidently held the study in high regard himself, since in 1955 (a year before his death) he had planned an English translation of it.

The article here translated exhibits the sensitivity to historical text and high regard for symbolic logic characteristic of his later work (cf. especially *Aristotle's Syllogistic*). Yet in a number of respects it reflects a highly developmental stage in his conception of logic. I will mention just three ways in which this emerges. First, Lukasiewicz seems unaware of any significant distinction between the algebra of Boole and the calculus of propositions. While this may seem strange in view of Frege's publication of the *Begriffschrift* in 1879, it is understandable in light of the fact that only with the publication in 1910 of *Principia Mathematica* did Frege's theory receive wide attention. Second, his suggestion (by analogy with the development of non-Euclidian geometries) that revision of basic laws of Aristotle's logic might yield new, non-Aristotelian logics suggests that as early as 1910 Lukasiewicz had conceived of the possibility of multi-valued logics. And, indeed, rejection of the Aristotelian principle of the excluded middle would enable generation of such logical systems. Third, two of his

traditional formal logic and especially the logic of Aristotle, modern symbolic logic points to and signifies an improvement similar in kind to that of modern geometry over Euclid's elements.[2] Just as in the course of the nineteenth century a more exact examination of the Euclidian parallel line postulate has led to new, non-Euclidian systems of geometry, so the conjecture would not be entirely out of order that a fundamental revision of basic laws (*Grundgesetze*) of Aristotle's logic might perhaps lead to new non-Aristotelian systems of logic. And even if the Aristotelian principles of logic were confirmed for all time, they nonetheless offer the modern investigator a wealth of unsolved problems. Above all there arises the question of how the highest of the basic laws of logic, whose number has substantially increased since Aristotle, should be formulated; and then in what relation do they stand to one another, especially whether they are independent of each other or whether in some way they lead back to a final principle; further whether their domain of validity (*Geltungsbereich*) is unrestricted or whether in fact certain exceptions

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reasons for claiming the indemonstrability of the principle of contradiction would probably have been gladly reformulated by Lukasiewicz in the light of logical developments after 1910. In section *19,a*, he suggests that we cannot know *a priori* that a constructive object is free of contradiction, citing the Russell paradox as a case. But the understanding and solution of such logical paradoxes has advanced considerably beyond that of 1910 and Lukasiewicz would undoubtedly have granted the need for reformulation of *19,a*. Then, at the end of section *18*, he states that the principle of contradiction is not required for direct (affirmative) proofs but only for indirect (negative) ones. This no longer seems tenable. For if we stipulate a system consisting of affirmative propositions only and within which deductive operations can occur, then in such a system it will not be possible to construct a proposition corresponding to the principle of contradiction (because we cannot express negation in the system). But the question of consistency of the system must be met. Since the system can be shown to be consistent if there is one proposition which is meaningful but unprovable in terms of the system, then the system will be consistent without direct recourse to negation. This consistency requirement is, however, something of a meta-logical correlate to the logical principle of contradiction. So the principle still enters in but in a different guise. Such post-1910 meta-logical considerations would doubtlessly have made Lukasiewicz amenable to altering his result in section *18*. (For the last point I am indebted to B. Sobocinski, *Philosophical Studies*, VI, [1956].)

[2] Italicizing corresponds throughout to Lukasiewicz' own emphasis.

are admitted; and finally, what gives us the justification to hold these basic laws as irrefutably true? These are candid questions which have indeed been occasionally raised and discussed before, but which are significantly more sharply formulated from the standpoint of symbolic logic and which can thus be put into a new light.

In the discussion at hand I have attempted to pave the way for such a treatment of the principle of contradiction. In a number of respects it seems to me worthwhile to relate my critical exposition to Aristotle's train of thought. Indeed, every critique must be raised against something substantial, otherwise it generally becomes the critic's leisurely game with his own cerebral phantasies. Now Aristotle's intuitions regarding the principle of contradiction are, for the most part and clear down to the present day, the usual and traditional ones; and arguments for and against the principle can be found together in the Stagirite in greater completeness than in any one modern textbook of logic. My investigation will proceed with the Aristotelian text at hand and with regard to the results of symbolic logic. The most important results shall be sketched very briefly in the following.

1. Aristotle formulates the principle of contradiction in a three-fold way, as an *ontological*, *logical*, and *psychological* law, without making explicit in any way the difference among them.

(a) Ontological formulation: *Met.* Γ 3. 1005b 19, 20: τὸ γὰρ αὐτὸ ἅμα ὑπάρχειν τε καὶ μὴ ὑπάρχειν ἀδύνατον τῷ αὐτῷ καὶ κατὰ τὸ αὐτὸ.—"It is impossible that the same thing belong and not belong to the same thing at the same time and in the same respect."

(b) Logical formulation: *Met.* Γ 6. 1011b 13, 14: ...βεβαιοτάτη δόξα πασῶν τὸ μὴ εἶναι ἀληθεῖς ἅμα τὰς ἀντικειμένας φάσεις.—"The most certain of all basic principles is that contradictory propositions (*Aussagen*) are not true simultaneously."

(c) Psychological formulation: *Met.* Γ 3. 1005b 23, 24: ἀδύνατον γὰρ ὀντινοῦν ταὐτὸν ὑπολαμβάνειν εἶναι καὶ μὴ εἶναι... —"No one can believe that the same thing can (at the same time) be and not be."

2. One could attempt to express these principles more precisely in the following way:

(a) Ontological, respectively "object-theoretical" ["*gegenstandstheoretische*"] formulation: *To no object can the same characteristic belong and not belong at the same time.* By "object," I understand with Meinong everything that is "something" and not "nothing"; by "characteristic," I mean everything which can be predicated of an object.

(b) Logical formulation: *Two conflicting (contradictory) propositions cannot be true at the same time.* By "proposition" [*Aussage*] I understand a string of words or other sensibly perceptible symbols whose meaning consists in the fact that they attribute or deny some characteristic to an object.

(c) Psychological formulation: *Two acts of believing which correspond to two contradictory propositions cannot obtain in the same consciousness.* By "act of believing" [*Glaubensakt*] I understand a psychical [*psychische*] function *sui generis*, which is also designated by the words "conviction" [*Überzeugung*], "recognition" [*Anerkennung*], "belief," etc., and which cannot be more finely explained but must rather be experienced.

3. The above formulation might also agree with those of Aristotle insofar as in a very similar way the Stagirite also often separates, on the one hand, the ontological or object-theoretical meaning of a proposition and, on the other hand, the psychical function of believing corresponding to the proposition. And certainly:

(a) Propositions (ἀπόφανσις = κατάφασις, affirmation, or ἀπόφασις, negation) according to Aristotle, indicate the fact(s) that something is or is not, i.e., they indicate the being or not being (τὸ εἶναι ἢ μὴ εἶναι), and, eventually, the being-so [*Sosein*] and not being-so of objects. Such facts have recently been called "objective facts" by Meinong (states of affairs [*Sachverhalte*] by Stumpf). So, in general, propositions indicate the fact that an object has or does not have a characteristic (being or being-so).

(b) According to Aristotle, assertions are sensibly perceptible symbols of psychical acts of believing (ὑπόληψις, sometimes also δόξα).

The places in *De Interpretatione* where Aristotle explains the concept of the assertion are conclusive on the point that assertions indicate objective facts: *De Interpr.* c. 4. 17a 1-3: ἐστι δὲ λόγος

ἅπας μὲν σημαντικός..., ἀποφαντικὸς δὲ οὐ πᾶς, ἀλλ' ἐν ᾧ τὸ ἀληθεύειν ἢ ψεύδεσθαι ὑπάρχει.—c. 1. 16a 16-18: καὶ γὰρ ὁ τραγέλαφος σημαίνει μέν τι, οὔπω δὲ ἀληθὲς ἢ ψεῦδος, ἐὰν μὴ τὸ εἶναι ἢ μὴ εἶναι προστεθῇ.—"Every utterance has some meaning, but not every one is an assertion; rather those to which being-true or being-false apply.—For even the goat-stag indicates something meaningful, but neither something true nor false, as long as being [*das Sein*] or not-being [*Nichtsein*] cannot be applied."

That assertions are symbols of acts of believing can be seen clearly from the following passage: *De Interpr.*, c. 14, 24b 1-3: ὥστε εἴπερ ἐπὶ δόξης οὕτως ἔχει, εἰσὶ δὲ αἱ ἐν τῇ φωνῇ καταφάσεις καὶ ἀποφάσεις σύμβολα τῶν ἐν τῇ ψυχῇ, δῆλον ὅτι καὶ καταφάσει ἐναντία... ἀπόφασις...—"If, therefore, the acts of believing behave in this way (i.e., if the affirming acts are antithetically opposed to the negating ones) and *if the linguistic affirmations and negations are symbols of psychical processes*, then clearly the (linguistic) affirmation is also antithetically opposed to the negation."

4. None of the three formulations of the principle of contradiction is identical in meaning with the others, for each contains expressions which designate essentially different objects (e.g., object and characteristic, assertion and true [*wahr*], belief, act, and consciousness, etc.). In contrast, the logical formulation seems to be for Aristotle logically equivalent with the ontological formulation.[3] The traditional [*althergebrachte*], even if deficiently formulated dictum: *veritas est adaequatio rei et intellectus* is rendered much more precisely by the Stagirite in the following way: *Met.* Γ 7. 1011b 26, 27: τὸ... γὰρ λέγειν..., τὸ ὂν εἶναι καὶ τὸ μὴ ὂν μὴ εἶναι ἀληθές...—"To say of that which is that it is and of that which is not that it is not is true."

The equivalence of the logical and the ontological principle of contradiction comes necessarily from the one-one correlation between assertions [propositions] and objective facts.

5. Aristotle attempts to prove the psychological principle of contradiction on the basis of the logical principle. The proof falls into two parts:

[3] Cf. *An. Pr.* A 46, 52a 32: τὸ γὰρ ἀληθές τῷ ἔστιν ὁμοίως τάττεται.

(a) *Met.* Γ 3. 1005b 26-32: εἰ δὲ μὴ ἐνδέχεται ἅμα ὑπάρχειν τῷ αὐτῷ τἀναντία..., ἐναντία δ'ἐστὶ δόξα δόξῃ ἡ τῆς ἀντιφάσεως, φανερόν ὅτι ἀδύνατον ἅμα ὑπολαμβάνειν τὸν αὐτὸν εἶναι καὶ μὴ εἶναι τὸ αὐτό. ἅμα γὰρ ἂν ἔχοι τὰς ἐναντίας δόξας ὁ διεψευσμένος περὶ τούτου. —"If it is not possible that to one and the same object antithetically opposed characteristics apply; and if two acts of believing, to which antithetically opposed propositions correspond, are themselves antithetically opposed; then clearly no one can believe at the same time that the same thing is and is not. For at the same time this one, who would here be in error, would have had antithetically opposed acts of believing."—On my view the rather difficult passage ἐναντία δ'ἐστί δόξα δόξῃ ἡ τῆς ἀντιφάσεως is to be interpreted as indicated above in accordance with the parallel passage in the final chapter of *De Interpretatione*: δόξα ἡ τῆς ἀποφάσεως, δόξα ἡ τοῦ ἐναντίου = ἡ τὸ ἐναντίον εἶναι δοξάζουσα (*De Interpr.* c. 14. 23a 27-39).

(b) *Met.* Γ 6. 1011b 15-21: ἐπεὶ δ'ἀδύνατον τὴν ἀντίφασιν ἀληθεύεσθαι ἅμα κατὰ τοῦ αὐτοῦ, φανερὸν ὅτι οὐδὲ τἀναντία ἅμα ὑπάρχειν ἐνδέχεται τῷ αὐτῷ. τῶν μὲν γὰρ ἐναντίων θάτερον στέρησίς ἐστιν οὐχ ἧττον, οὐσίας δὲ στέρησις. ἡ δὲ στέρησις ἀπόφασις ἐστιν ἀπό τινος ὡρισμένου γένους. εἰ οὖν ἀδύνατον ἅμα καταφάναι καὶ ἀποφάναι ἀληθῶς, ἀδύνατον καὶ τἀναντία ὑπάρχειν ἅμα...—"If it is impossible to truthfully assert contradictory characteristics at the same time of one and the same object, then it is obvious that antithetically opposed characteristics cannot hold of one and the same object simultaneously. For of two antithetically opposed characteristics the one is just as much privation as the other, namely, privation of being; the privation, however, is negation of a determinate species. Thus, if it is impossible to truthfully affirm and deny something simultaneously, it is also impossible that antithetically opposed characteristics hold of the same object."

Precisely formulated the Aristotelian proof of the psychological principle of contradiction reads as follows:

Were it possible that two acts of believing, corresponding to contradictory assertions, could obtain in the same consciousness, then antithetically opposed characteristics would hold of this consciousness at the same time. But on the basis of the logical principle of contradiction, it is impossible that incompatible char-

acteristics hold of the same object at the same time. It follows that two acts of believing corresponding to contradictory assertions [propositions] cannot obtain in the same consciousness at the same time.[4]

6. Aristotle's proof of the psychological principle of contradiction is incomplete because Aristotle did not demonstrate that acts of believing which correspond to contradictory propositions are incompatible. Discussions related to this point are found in the final chapter of *De Interpretatione*.[5] However, they are inconclusive for two reasons:

(a) Antithetically opposed [*konträr-entgegengesetzt*] means for Aristotle characteristics which lie farthest apart from each other in a series (e.g., "black" and "white" in the series "colorless" colors). Every series must be constructed on the basis of an *ordering relation*. Aristotle adopts as the ordering relation of acts of believing differences in their degree of being true or being false, and he even speaks of "truer" and "falser" beliefs (*De Interpr.*, c. 14. 23b 17. μᾶλλον ἀληθὴς *scil.* δόξα, 20. μᾶλλον ψευδὴς δόξα). *It is, however, impossible that there be differences in degree of being true or false.*

(b) In the psychological investigation of acts of believing (*De Interpr.*, c. 14) Aristotle commits the very common fallacy of "logicism in psychology," which can pass for the counterpart of "psychologism in logic." Instead of investigating psychical functions, the Stagirite considers the propositions corresponding to them and their *logical* relations. That is shown:

(a′) In that he characterizes acts of believing as true or false, although as psychical functions, acts of believing could be true or false in the primary sense no more than could sensations, feelings, and the like. "True" and "false" are relative characteristics which

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[4] I agree completely with Maier on the interpretation of the passage just quoted (cf. *Die Syllogistik des Aristoteles*, I, [Tübingen, 1896], p. 45). I am indebted generally to Maier's basic and worthwhile work for much historical advice.

[5] Even Alexander of Aphrodisias refers to this: ὅτι δὲ ἐναντίαι αἱ δόξαι τῆς ἀντιφάσεως δέδεικται διὰ πλειόνων ἐπὶ τέλει τοῦ περὶ Ἑρμηνείας *(Scholia in Aristotelem*, Brandis, ed. Acad. Bor., p. 652).

belong only to assertions as representations of the objective [*Abbildungen der Objective*].

(b′) Further, Aristotle confuses [*vermengt*] logical succession [*logische Abfolge*] with psychical causality. Characteristic of this is the passage at *De Interpr.*, c. 14. 23b 25-27: ἡ δὲ(*scil.* δόξα ἡ τῆς ἀποφάσεως) τοῦ ὅτι κακὸν τὸ ἀγαθὸν συμπεπλεγμένη ἐστί. καὶ γὰρ ὅτι οὐκ ἀγαθὸν ἀνάγκη ἴσως ὑπολαμβάνειν τὸν αὐτόν. —"The conviction that the good is not good is closely intertwined with the conviction that the good is bad; for (whoever holds the good to be bad) the same one must believe as well that the good is not good."—Indeed, if he only *thinks* about it, and that must he not, and whether it were even possible at all to have such perverse "convictions"!

7. Regardless of Aristotle's reasoning, the following can be said about the psychological principle of contradiction:

(a) The psychological principle of contradiction cannot be demonstrated *a priori*, rather it is at most to be induced as *a law of experience*.

(b) The principle in question has not yet been *empirically* demonstrated.[6]

(c) It is questionable whether it is provable at all. In any event there are sufficient examples in the history of philosophy where contradictions have been asserted at the same time and with full awareness.[7] In order, then, to save the principle in the face of this, one would have to resort to *supporting hypotheses*, to which even Aristotle occasionally takes recourse (cf. *Met.* Γ 3. 1005b

[6] It would not be out of place to recall once again the barbed but appropriate remarks of Husserl (*Logische Untersuchungen*, vol. I, [Halle, 1900], p. 82): "In the same individual, or still better, in the same consciousness, contrary acts of believing could never persist during even the smallest interval of time. But is this really a law? May we really state it with unlimited generality? Where are the psychological inductions which justify its adoption? Might there not have been and might there not be men, who confused by fallacies for instance, occasionally held opposites to be true simultaneously? Has scientific research been conducted as to whether something like this does not occur among the insane and perhaps even in plain contradictions? How does the hypothesis fare with the conditions of fever delirium, etc.? Is the law also valid for animals?"

[7] As elaboration, the following passage from Hegel might be adduced (*Wissenschaft der Logik*, Werke, vol. IV, [Berlin, 1834], p. 69): "Some-

25, 26: οὐκ ἐστι γὰρ ἀναγκαῖον ἅ τις λέγει ταῦτα καὶ ὑπολαμβάνειν.—"For one does not also have to believe that which he says." But supporting hypotheses detract from the degree of probability [*Wahrscheinlichkeitswert*] due the basic thesis.

The psychological formulation of the principle of contradiction must, therefore, be excluded from further investigation as a thesis of questionable worth which is to be proven empirically but as yet remains unproved.

8. Aristotle considers the ontological, and respectively, the logical principle of contradiction a final, unprovable law.—However, *he does not prove this claim.* Rather it is limited only to the hint that "if one may not demand proof for something, it would also not be easy to say which of the principles should be taken as prior" (*Met.* Γ 4. 1006a 10, 11: εἰ δέ τινων μὴ δεῖ ζητεῖν ἀπόδειξιν, τίνα ἀξιοῦσιν εἶναι μᾶλλον τοιαύτην ἀρχὴν οὐκ ἂν ἔχοιεν εἰπεῖν).

9. In reference to this it must first be emphasized that there are "simpler" and "more evident" principles, which could hold good as prior to the principle of contradiction as a final and unprovable principle. Above all, the *principle of identity* belongs here. It reads: To each object belongs that characteristic to which it belongs.

(a) The principle of identity is *different* from the law of contradiction. The principle of contradiction cannot be formulated without the concepts of *negation* and *logical multiplication*, which are expressed in the words "and at the same time"; while the principle of identity holds very well without recourse to those concepts.

(b) Symbolic logic first assisted us toward clarity in this question. The so-called "philosophical" logic is in this light nothing more than bold phraseology. The *principium identitatis* was understood at one time as the principle of identity, at another time as the principle of contradiction [8]; the principle of contradiction was confused with the deficiently formulated *principle of*

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thing moves itself not merely insofar as it is here in this instant and there in another instant, rather insofar as it is in one and the same instant here and not here and insofar as in this here it both is and is not."

[8] Cf. Trendelenburg, *Logische Untersuchungen*, I (Leipzig, 1862), p. 31, and Sigwart, *Logik* (Freiburg i. B., 1889), I, p. 186.

double negation, "A is not not-A"; the principle of identity, for which the highly ambiguous and at the least imprecise formula "A is A" was generally employed (is the "divisible by two" divisible by two?), was placed to the side as a "positive counterpart" [9] [*positive Kehrseite*] of the principle of contradiction and identified with it, etc. The philosophical logic simply had no appreciation for the finer conceptual distinctions because it did not operate with sharply delineated concepts and unambiguously determined symbols; rather it sank into the swamp of the fluid and vague speech used in everyday life.

10. But not even the principle of identity is an ultimate law, for it can be demonstrated in terms of the definition of true propositions. In general, one might attempt to set down the following basic laws:

(a) All *a priori* principles must be demonstrable and must be demonstrated [*proven*].

(b) There is only one principle which cannot be demonstrated in terms of other principles but which is rather true and demonstrated "through itself" [*durch sich selbst*]. This is the proposition:

"An affirmative proposition I designate as true, when it confers on an object the characteristic appropriate to it."

This proposition is affirmative and confers on me a characteristic which must certainly fall to me, namely, the characteristic that I designate as "true" propositions constituted so and so. For, it is certain fact that I do so designate at the moment in which I utter or write the proposition under discussion. The explanation of what I understand by a true proposition is thus "through itself" true and demonstrated.

(c) Every other *a priori* basic law, even the principle of contradiction, must be derived from previously demonstrated principles, if it is to count as true.

11. Although Aristotle proclaims the nondemonstrability of the principle of contradiction, he strives in spite of that to give demonstrations for the principle. *Met.* Γ 4. 1006a 11-13: ἔστι δ'ἀποδεῖξαι ἐλεγκτικῶς καὶ περὶ τούτου ὅτι ἀδύνατον, ἐάν μόνον τι λέγῃ

[9] Cf. Sigwart, *op. cit.*

ὁ ἀμφισβητῶν.—"However, an elenctic demonstration is also available here (namely, the impossibility that contradictory assertions can be true at the same time), if only the opponent says something."—Now there lies in this a contradiction, which is only apparently hidden by the word "elenctic" (ἐλεγκτικῶς), but which can in no way be avoided.

(a) By "elenchus" Aristotle understands a syllogism, which contains the contradictory opposite of a prior thesis. (Cf. *An. Pr.*, B, 66b 11: ὁ γὰρ ἔλεγχος ἀντιφάσεως συλλογισμός). If, for instance, one held that the principle of contradiction did not hold (thesis), and is then compelled to accept such premisses from which the truth of this principle (namely, the contradictory opposite of the previous thesis) follows with syllogistic necessity, then such a syllogism is respectively called "elenctic" demonstration. Thus, according to Aristotle, the *elenchus is a rule-governed inference*, which is only superficially distinguished from genuine demonstration and precisely so in that it is used immediately as a refutation.

(b) As opposed to this, the proposed distinction between genuine and elenctic proof of the law of contradiction, which is offered in *Met.* Γ 4, appears as a vacuous phrase of embarrassment [*nichtssagende Verlegenheitsphrase*]: *Met.* Γ 4. 1006a 15-18: τὸ δ'ἐλεγκτικῶς ἀποδεῖξαι λέγω διαφέρειν καὶ τὸ ἀποδεῖξαι, ὅτι ὁ ἀποδεικνύων μὲν ἂν δόξειεν αἰτεῖσθαι τὸ ἐν ἀρχῇ, ἄλλου δὲ τοῦ τοιούτου αἰτίου ὄντος ἔλεγχος ἂν εἴη καὶ οὐκ ἀπόδειξις.—"I distinguish, however, elenctic demonstration from genuine, because were the demonstration to appear to commit a *petitio principii* but another be guilty thereof (i.e. of the *petitio principii*), then an elenctic demonstration would be possible, but not a genuine one."—The sense of this passage appears to me to be: Whoever wants to demonstrate the law of contradiction commits the fallacy of *petitio principii* and the demonstration is false. If, however, another is guilty of making this mistake, then an elenchus is possible—and everything is in order. I cannot grasp what is being said here.

(c) Both of the first Aristotelian proofs of the principle of contradiction actually accord—at least in their intention—with the definition of elenctic demonstration given in the *Analytics*. Aristotle concludes the proofs with the words: *Met.* Γ 4. 1007b 17-18: εἰ δὲ τοῦτο, δέδεικται ὅτι ἀδύνατον ἅμα κατηγορεῖσθαι τὰς

ἀντιφάσεις.—"When it is thus the case, then *the evidence* [*Nachweis*] *is supplied* that contradictions cannot possibly be asserted at the same time."

(d) Aristotle demonstrates the principle of contradiction not only elenctically but also *ad impossibile* [*apagogisch*]. However, *ad impossibile* demonstrations presuppose that principle and, consequently, contain a *petitio principii* in case they are used for the purpose of proving it.

It is entirely clear from the above observations that Aristotle commits no contradiction when, on the one hand, he declares the principle of contradiction to be nondemonstrable and, on the other hand, attempts to demonstrate the same principle elenctically and *ad impossibile*.

12. Aristotelian demonstrations of the principle of contradiction:

The presupposition, whose recognition is to be forced from the opponent, of the elenctic demonstration: Let a word be given which signifies something essentially singular [*das etwas in seinem Wesen Einheitliches bedeute*]. For example, let the word "man" be given and let it signify a two-legged living creature.

(a) The first elenctic proof: *Met.* Γ 4. 1006b 28-34: ἀνάγκη τοίνυν, εἴ τι ἔστιν ἀληθὲς εἰπεῖν, ὅτι ἄνθρωπος, ζῷον εἶναι δίπουν· τοῦτο γὰρ ἦν ὅ ἐσήμαινε τὸ ἄνθρωπος· εἰ δ'ἀνάγκη τοῦτο, οὐκ ἐνδέχεται μὴ εἶναι τὸ αὐτὸ ζῷον δίπουν· τοῦτο γὰρ σημαίνει τὸ ἀνάγκη εἶναι, τὸ ἀδύνατον εἶναι μὴ εἶναι. οὐκ ἄρα ἐνδέχεται ἅμα ἀληθὲς εἶναι εἰπεῖν τὸ αὐτὸ ἄνθρωπον εἶναι καὶ μὴ εἶναι ἄνθρωπον.—"If one can truly say of something that it is man, it is necessary that it be a two-legged living creature; for it was that which the word "man" signified. If, however, this is necessary, so it is impossible that the same thing not be a two-legged creature. For necessity means just the impossibility of not being. Accordingly, it is not possible to assert at the same time that the same thing is man and is not man (respectively, two-legged living creature)."

Formulated generally and precisely, this proof reads as follows: With the word A, I signify something which is in its essence B. Consequently, the object A is necessarily a B. If, however, A is necessarily a B, so it cannot—by reason of the

meaning of the word "necessarily"—possibly not be *B*. Accordingly, no *A* can simultaneously be and not be *B*.

(b) The second elenctic proof: *Met.* Γ 4. 1006b 11-22: ἔστω δὴ, ..., σημαῖνον τι τὸ ὄνομα καὶ σημαῖνον ἕν. οὐ δὴ ἐνδέχεται το ἀνθρώπῳ εἶναι σημαίνειν ὅπερ ἀνθρώπῳ μὴ εἶναι, εἰ τὸ ἄνθρωπος σημαίνει ... ἕν ... καὶ οὐκ ἔσται εἶναι καὶ μὴ εἶναι τὸ αὐτὸ ἀλλ' ἢ καθ' ὁμωνυμίαν, ὥσπερ ἂν εἰ ὃν ἡμεῖς ἄνθρωπον καλοῦμεν, ἄλλοι μὴ ἄνθρωπον καλοῖεν· τὸ δ'ἀπορούμενον οὐ τοῦτό ἐστιν, εἰ ἐνδέχεται τὸ αὐτὸ ἅμα εἶναι καὶ μὴ εἶναι ἄνθρωπον τὸ ὄνομα, ἀλλὰ τὸ πρᾶγμα.—"Suppose a word to be given which signifies something and in particular something singular. Then it is not possible that being a man [τὸ ἀνθρώπῳ εἶναι] means the same as not being a man, so far as the word "man" signifies something singular [ἕν]. Consequently, one and the same thing can be and not be only homonymously, as when that which we call man others want to call not man. But the point does not turn on whether one and the same thing can be *named* man simultaneously, but whether it can *be* so."

Generally and precisely formulated, this proof reads as follows: With the word *A*, I signify something which is in its essence *B*. Consequently, the object *A*, which is in its essence *B*, cannot in its essence at the same time be not-*B*, for otherwise it would not be unified in its essence. Accordingly, *A* cannot simultaneously be and not be *B*.

The three most important of the *ad impossibile* proofs may be introduced:

(c) The first proof *ad impossibile*: *Met.* Γ 4. 1007b 18-21: ἔτι εἰ ἀληθεῖς αἱ ἀντιφάσεις ἅμα κατὰ τοῦ αὐτοῦ πᾶσαι, δῆλον ὡς ἅπαντα ἔσται ἕν. ἔσται γὰρ τὸ αὐτὸ καὶ τριήρης καὶ τεῖχος καὶ ἄνθρωπος . . .—

—"Further, if all contradictory propositions were true at the same time in respect to the same thing, then clearly everything will be one. For a trireme, a wall, and a man would then be the same."

(d) The second proof *ad impossibile*: *Met.* Γ 4. 1008a 28-30: πρὸς δὲ τούτῳ ὅτι πάντες ἂν ἀληθεύοιεν καὶ πάντες ἂν ψεύδοιντο, καὶ αὐτὸς αὑτὸν ὁμολογεῖ ψεύδεσθαι.—"Beyond this, it follows that everyone speaks the true and the false and must admit that he speaks the false."

(e) The third proof *ad impossibile*: *Met.* Γ 4. 1008b 12-19: ὅθεν καὶ μάλιστα φανερόν ἐστιν ὅτι οὐδεὶς οὕτω διάκειται οὔτε τῶν ἄλλων οὔτε τῶν λεγόντων τὸν λόγον τοῦτον. διὰ τί γὰρ βαδίζει Μέγαράδε ἀλλ' οὐχ ἡσυχάζει οἰόμενος βαδίζειν; οὐδ' εὐθέως ἔωθεν πορεύεται εἰς φρέαρ ἢ εἰς φάραγγα, ἐάν τύχῃ, ἀλλὰ φαίνεται εὐλαβούμενος, ὡς οὐχ ὁμοίως οἰόμενος μὴ ἀγαθὸν εἶναι τὸ ἐμπεσεῖν καὶ ἀγαθόν; δῆλον ἄρα ὅτι τὸ μὲν βέλτιον ὑπολαμβάνει τὸ δ' οὐ βέλτιον.—"From that, one can clearly see that no one believes such a thing, neither anybody else nor one who practices such rhetoric. For why does such a one still go to Megara instead of quietly sitting at home with the thought that he is going? Or why does he not one fine morning immediately throw himself into the well or abyss, when it is directly come upon; rather he takes care, as if he is of the opinion that falling in is not equally good and not good."

13. Criticism of the Aristotelian proofs of the principle of contradiction.

(a) The first elenctic proof is inadequate because what is proved by it is not the principle of contradiction but at most the *principle of double negation*: If something is B, then it cannot be not-B. However:

(a′) the principle of double negation is different from the principle of contradiction because—as symbolic logic has shown —it can be very nicely expressed without the notion of *logical multiplication*, while the principle of contradiction would not stand failing this notion.

(b′) There are objects, namely contradictory ones (e.g., "the greatest prime number"), for which the principle of double negation is valid, but not the principle of contradiction. Hence, an inference concerning the principle of contradiction cannot be made from the principle of double negation.

(b) The second elenctic proof is inadequate because

(a′) in the most favorable circumstances it would establish the principle of contradiction for a very limited range of objects only, namely, for the "essence" of things or for substance. For accidents, its validity would still be questionable. The fact that in this proof Aristotle vindicates the principle of contradiction for substances is yielded, for example, from the following passage: *Met.* Γ 4. 1007b 16-18: ἔσται ἄρα τι καὶ ὡς οὐσίαν σημαῖνον. εἰ δὲ

τοῦτο, δέδεικται ὅτι ἀδύνατον ἅμα κατηγορεῖσθαι τὰς ἀντιφάσεις.—"And so there will also be something given which signifies substance. But if this is so, then the proof is at hand that contradictories cannot possibly be jointly predicated."

(b′) The existence of substances is only *probable*. Consequently, the principle of contradiction can also be accepted only as *probable*, insofar as it relates to substances.

(c′) The proof contains a *formal mistake* because it uses a premiss which is demonstrated *ad impossibile* only: If in its essence an object could be and not be *B* simultaneously, then it would not be unitary; *B* is namely something other than not-*B*. But proofs *ad impossibile* presuppose the principle of contradiction.

(ad c, d, e) All the proofs *ad impossibile* are inadequate because they contain the following two *formal mistakes*:

(a′) A *petitio principii* is contained in each. The *ad impossibile* mode of inference turns namely on the principle of contraposition which—as symbolic logic has shown—presupposes the principle of contradiction. This can also be put into words: The *ad impossibile* mode of inference runs: If *a* is, then *b* must be; now *b* is not; thus, *a* also cannot be. Reason: Were *a* to be, then a contradiction would ensue, for *b* must also be, which it is not.

(b′) All of Aristotle's proofs *ad impossibile* meet the objection of the *ignoratio elenchi*. Aristotle proves not that the mere *denial* of the principle of contradiction would lead to absurd consequences, rather he attempts to establish the impossibility of the assumption that *everything* is contradictory. One sees this quite clearly, for example, from the observation (cf. above 12c): "if *all* contradictory predications are simultaneously true, etc."—However, he who denies the principle of contradiction or who demands a proof for it, surely does not need to accept that *everything* is contradictory, especially those processes and facts which determine practical affairs.

It is clear from the above considerations that, in spite of concerted effort, Aristotle has not proven the principle of contradiction.

14. The just mentioned *shift in the point of the proof* [*Verrückung des Beweispunktes*] must be specially pointed out. Besides the passage already introduced, *Met.* Γ 4, 1007b 19, these other following passages are important in this context: *Met.* Γ 4, 1006a 29-31, 1008a 8-16, 1008b 31, 1009a 5 (end of *Met.* Γ 4). The last passage is particularly characteristic of Aristotle's exposition: ἔτι εἰ ὅτι μάλιστα πάντα οὕτως ἔχει καὶ οὐχ οὕτως, ἀλλὰ τό γε μᾶλλον καὶ ἧττον ἔνεστιν ἐν τῇ φύσει τῶν ὄντων· οὐ γὰρ ἂν ὁμοίως φήσαιμεν εἶναι τὰ δύο ἄρτια καὶ τὰ τρία, οὐδ' ὁμοίως διέψευσται ὁ τὰ τέτταρα πέντε οἰόμενος καὶ ὁ χίλια. εἰ οὖν μὴ ὁμοίως, δῆλον ὅτι ἄτερος ἧττον, ὥστε μᾶλλον ἀληθεύει. εἰ οὖν τὸ μᾶλλον ἐγγύτερον, εἴη γ' ἄν τι ἀληθὲς οὗ ἐγγύτερον τὸ μᾶλλον ἀληθές. κἂν εἰ μή ἐστιν, ἀλλ' ἤδη γέ τι ἐστὶ βεβαιότερον καὶ ἀληθινώτερον, καὶ τοῦ λόγου ἀπηλλαγμένοι ἂν εἴημεν τοῦ ἀκράτου καὶ κωλύοντός τι τῇ διανοίᾳ ὁρίσαι. —"Further, *even if it were the case that everything is very much so and not so*, there still is a more or a less which is grounded in the nature of things. For we would not call two and three even in the same way, and he who holds four to be five does not err in the same way as he who holds four to be a thousand; the one clearly errs less and, therefore, expresses something more true. Now, if that which is more true is nearer the truth, there must also exist an (absolute) truth with respect to which that which is more true is nearer. And even if it doesn't exist, there is at least something which is (relatively) more certain and more true, and so we would be exempt from that senseless discourse which admits of no logical determination of a thing."

One sees from this most clearly that at the end of his exposition the Stagirite's task is no longer to prove the principle of contradiction in its generality, but rather to at least find an absolute and contradiction-free truth which would establish the falsity of the thesis antithetically opposed to the principle of contradiction: for every object, "the same characteristic belongs to and does not belong to it at the same time."

15. This note-worthy and yet, in its historical importance, unhonored *shift of proof* has good reason in certain of Aristotle's positive convictions.

(a) In one of the passages most important for the principle of contradiction the Stagirite does not appear to have aligned him-

self directly against the views of the sensualists. The following passage from *Met.* Γ 4 may be introduced: 1009a 22-36: ἐλήλυθε δὲ τοῖς διαποροῦσιν αὕτη ἡ δόξα ἐκ τῶν αἰσθητῶν, ἡ μέν τοῦ ἅμα τὰς ἀντιφάσεις καὶ τἀναντία ὑπάρχειν, ὁρῶσιν ἐκ ταὐτοῦ γιγνομένα τἀναντία . . . πρὸς μὴν οὖν τοὺς ἐκ τούτων ὑπολαμβάνοντας ἐροῦμεν, ὅτι τρόπον μέν τινα ὀρθῶς λέγουσι, τρόπον δέ τινα ἀγνοοῦσιν. τὸ γὰρ ὂν λέγεται διχῶς, ὥστ' ἔστιν ὂν τρόπον ἐνδέχεται γίγνεσθαί τι ἐκ τοῦ μὴ ὄντος, ἔστι δ' ὂν οὔ, καὶ ἅμα τὸ αὐτὸ εἶναι καὶ ὂν καὶ μὴ ὄν, ἀλλ' οὐ κατὰ ταὐτὸ ὄν, δυνάμει μὲν γὰρ ἐνδέχεται ἅμα ταὐτὸ εἶναι τὰ ἐναντία, ἐντελεχείᾳ δ' οὔ.—"Those who see an actual difficulty here have arrived at their view (that contradictory and antithetically opposed characteristics can obtain simultaneously) on the basis of sensible perception in that they notice that from one and the same thing proceed contraries. . . . To those so opinionated we answer that they are clearly right in one respect but reveal their ignorance in another respect. 'That which is' has namely two meanings, so that in one sense something can arise out of that which is not, in another sense not; and also the same thing can at once be that which is and that which is not, only not under the same meaning. *Potentially, the same thing can have antithetically opposed characteristics at the same time, but not actually.*"

First, it is of importance to establish that Aristotle limits the range of validity of the principle of contradiction to actual existents [*Seiende*] only.—We compare, then, the passage just introduced with the one following: *Met.* Γ 5. 1010a 1-5: αἴτιον δὲ τῆς δόξης τούτοις ὅτι περὶ τῶν ὄντων μὲν τὴν ἀληθείαν ἐσκόπουν, τὰ δ' ὄντα ὑπέλαβον εἶναι τὰ αἰσθητὰ μόνον· ἐν δὲ τούτοις πολλὴ ἡ τοῦ ἀορίστου φύσις ἐνυπάρχει, καὶ ἡ τοῦ ὄντος οὕτως ὥσπερ εἴπομεν. διὰ εἰκότως μὲν λέγουσιν, οὐκ ἀληθῆ δὲ λέγουσιν.—"The origin of this view (i.e., the view that at the same time it can be the case that things are so and not so, 1009b 32-33) lies, however, in this—that they (i.e., the sensualists) certainly investigated the truth concerning existing things, but took existing things to be sensibly perceptible ones only. *Here, however, the nature of the indeterminate predominates and that (potential) kind of being,* of which we have just spoken. Then, they speak convincingly, but they fail to state the (full) truth."

Consequently, according to Aristotle the sensibly perceptible

world, conceived as becoming and passing away, could *contain contradictions* as a strictly potential being. Indeed Aristotle did not have the courage to admit that openly, and merely makes diplomatic reference to an earlier passage, but the sense of his statement is completely unambiguous and is confirmed to the extent that for the Stagirite the indeterminate is precisely the potential. Cf. *Met.* Γ 4. 1007b 28, 29: τὸ γὰρ δυνάμει ὂν καὶ μὴ ἐντελεχείᾳ τὸ ἀόριστον ἐστιν.—"For what exists potentially and not actually is the indeterminate."

(b) In this light not only the above discussed shift of proof but also the meaning of the very important second elenctic proof becomes immediately clear: The ephemeral, sensibly perceptible world can contain contradictions, as many as it but wills; yet beyond it there is still another, eternal, and non-ephemeral world of *substantial essences*, which remains intact and shielded from every contradiction. The sensualists certainly are correct, but they fail to know the *whole* truth. And, therefore, Aristotle demands of them that they too "recognize another substance of existing things [*Substanz des Seienden*], which has neither change nor passing away nor creations," (*Met.* Γ 5. 1009a 36-38: ἔτι δ' ἀξιώσομεν αὐτούς ὑπολαμβάνειν καὶ ἄλλην τινὰ οὐσίαν τῶν ὄντων, ἣ οὔτε κίνησις ὑπάρχει οὔτε φθορὰ οὔτε γένεσις τὸ παράπαν.—Cf. also *Met.* Γ 5. 1010a 32-35).

Accordingly, it must be established that for Aristotle the principle of contradiction is to be thought of not as a general ontological law but rather as a metaphysical one, which is supposed to hold for substances primarily and with respect to which it is at least questionable whether its range of validity extends to appearances as well.[10]

16. Aristotle views the principle of contradiction not only as the most final [*das allerletzte*] but also as the *supreme* law.

[10] My interpretation of the Aristotelian principle of contradiction is thus essentially different from that of Maier (cf. *loc. cit.*, vol. I, p. 101). The fact, however, that Aristotle occasionally commits inconsistencies and in general is not always clear himself in this more difficult than usually accepted question, which was raised by him for the first time; this fact can to some degree justify interpretations of his thought which deviate from one another.

Met. Γ 3, 1005b 32-34: διὸ πάντες οἱ ἀποδεικνύντες εἰς ταύτην ἀνάγουσιν ἐσχάτην δόξαν· φύσει γὰρ ἀρχὴ καὶ τῶν ἄλλων ἀξιωμάτων αὕτη πάντων.—"Therefore, in point of providing a proof all must return to this principle as the final one; for this same one is the natural principle for all other axioms."

Now even according to Aristotle the principle of contradiction is not the highest law, at least not in the sense that *it yields a necessary presupposition for all other logical axioms.* In particular the *principle of the syllogism* is independent of the principle of contradiction. This is gotten from a long overlooked and misunderstood passage in the second Analytic:[11] *An. Post.* A11, 77a 10-22: τὸ δὲ μὴ ἐνδέχεσθαι ἅμα φάναι καὶ ἀποφάναι οὐδεμία λαμβάνει ἀπόδειξις, ἀλλ' ἢ ἐάν δέῃ δεῖξαι καὶ τὸ συμπέρασμα οὕτως. δείνυται δὲ λαβοῦσι τὸ πρῶτον κατὰ τοῦ μέσου, ὅτι ἀληθές, ἀποφάναι δ' οὐκ ἀληθές. τὸ δὲ μέσον οὐδὲν διαφέρει εἶναι καὶ μὴ εἶναι λαβεῖν, ὡς δ' αὕτως καὶ τὸ τρίτον. εἰ γὰρ ἐδόθη καθ' οὗ ἄνθρωπον ἀληθὲς εἰπεῖν, εἰ καὶ μὴ ἄνθρωπον ἀληθές, ἀλλ' εἰ μόνον ἄνθρωπον ζῷον εἶναι μὴ ζῷον δὲ μή. ἔσται γὰρ ἀληθὲς εἰπεῖν Καλλίαν, εἰ καὶ μὴ Καλλίαν, ὅμως ζῷον, μὴ ζῷον δ' οὔ. αἴτιον δ' ὅτι τὸ πρῶτον οὐ μόνον κατὰ τοῦ μέσου λέγεται ἀλλὰ καὶ κατ' ἄλλου διὰ τὸ εἶναι ἐπὶ πλειόνων, ὥστ' οὐδ' εἰ τὸ μέσον καὶ αὐτό ἐστι καὶ μὴ αὐτό, πρὸς τὸ συμπέρασμα οὐδὲν διαφέρει.—"The impossibility of joint affirmation and denial is presupposed by no proof (syllogism) unless the conclusion itself was also to have demonstrated such. Then it is demonstrated insofar as one accepts that it is true to predicate the major term of the middle term and not true to deny it. But as far as concerns the middle term and likewise the minor term, it makes no difference to hold that it is and is not. If, for instance, an object is given (e.g., Callias) of which one can truthfully predicate that it is man and insofar as man just is a living creature and not also not a living creature; so will it be true to predicate that Callias is a living creature and not also not a living creature, even if man were not man and Callias not Callias. The reason for this lies in the fact that the major term holds not only of the middle term but also of other objects as well

[11] Cf. Maier, *loc. cit.*, vol. II, p. 238, ff. 3 and I. Husic, "Aristotle on the Law of Contradiction and the Basis of the Syllogism," *Mind*, XV, (1906), pp. 215-222.

because it has a greater range (than the middle term); so that it makes no difference in the conclusion, if the middle term is the same and not the same."

According to Aristotle this syllogism is valid (A = living creature, B = man, C = Callias):

B is A (and not also not-A)
C, which is not-C, is B and not-B

C is A (and not also not-A).

However, if a syllogism remains valid when the principle of contradiction doesn't, then the principle of the syllogism (and indeed the *dictum de omni et nullo*) is independent of the principle of contradiction.

17. This conclusion is completely confirmed by modern symbolic logic. Beyond that, symbolic logic also shows that there are many other logical principles and theses which are independent of the principle of contradiction. The principle of identity, the basic principles of simplification and composition, the principle of distribution, the laws of tautology and absorption, and others would still continue to hold, even if the principle of contradiction no longer held.[12] Moreover, it would not be at all difficult to show in words, as well, that the basic principles of deduction as well as induction do not on the whole presuppose the principle of contradiction. Indeed there are innumerable deductions and inductions which proceed only by affirmative propositions; consequently, the principle of contradiction finds no application to these because it always meets an affirmative proposition and its contradictory negative.

On my view, *we must give up the false, though widely spread view that the principle of contradiction is the highest principle of all demonstrations!* That holds only for *indirect* proofs; for the *direct* ones, it is not true.

18. With that the historical-critical exposition is at an end. In the following positive part of the paper, I will attempt to state

[12] The clear and precisely formulated work by Couturat can serve as the best introduction to symbolic logic: *L'Algèbre de la Logique* ("Scientia," Phys-mathem., No. 24, [Paris, 1905]).

an opinion on the question of whence are we justified in holding the principle of contradiction as true.

(A) The principle of contradiction cannot be proven by proclaiming it *directly evident*. For:

(a′) evidence does not appear to be a permissible criterion of truth; it turns out that false propositions as well are held to be evident (cf. the Cartesian proof of God).

(b′) the principle of contradiction does not appear to be evident to everyone; for the old eristic thinkers of Megara or for Hegel it was in all probability not evident.

(B) The principle of contradiction cannot be proven by setting it up as a natural law determined by the *psychical organization* of man. For:

(a′) it is possible to determine false propositions by our psychical organization (cf., e.g., many sensory hallucinations);

(b′) it is questionable whether the principle of contradiction can be validated as a law determined by the psychical organization of man (cf. the remarks in 7 above regarding the psychological principle of contradiction).

(C) The principle of contradiction cannot be proven on the basis of the definition of false statements or negations. Sigwart [13] has suggested this means, but Aristotle already has this very proof in mind when he says: *Met.* Γ 4. 1008a 34-b 1: ἔτι εἰ ὅταν ἡ φάσις ἀληθὴς ᾖ, ἡ ἀπόφασις ψευδής, κἂν αὕτη ἀληθὴς ᾖ. ἡ κατάφασις ψευδής, οὐκ ἂν εἴη τὸ αὐτὸ ἅμα φάναι καὶ ἀποφάναι ἀληθῶς.—"Further, if the negation is false whenever the affirmation is true and the affirmation false whenever the negation is true, then one and the same thing cannot be jointly affirmed and denied."—But he immediately drops this proof because he believes "one could suspect a *petitio principii* in it" (1008b 1-2: ἀλλ' ἴσως φαῖεν ἂν τοῦτ' εἶναι τὸ ἐξ ἀρχῆς κείμενον).—Certainly this proof would not be a *petitio principii*, but nonetheless it is inadequate. For:

(a′) if one also accepts that the negation "A is not B" means the falsity of the affirmation "A is B," then the principle of contradiction is not to be deduced therefrom. The notion of *logical multiplication* is not contained in the definition of nega-

[13] *Logic*, vol. I, p. 182 ff.

tion, respectively falsity, and it is this notion which directly bestows on the principle of contradiction its characteristic imprint. Two contradictory propositions cannot be true simultaneously (affirmation and negation; truth and falsity contain each other [*heben einander auf*]) and cannot both be characteristic of the same object. In terms of the definition of falsity or negation, however, it would still be possible to accept that the assertion "A is B" and "A is not B" hold at the same time in that they are both true and false at the same time.

(b′) Of course if one prefers rather to avoid designating one and the same proposition as true and false, another definition of falsity can be set up which is of much greater account than the usual definition in terms of the basic thought in the concept, in that it is much more carefully formulated. The basic notion of falsity is, namely, *that false propositions are no representation of the objective*, or—in other words—*that false propositions correspond to nothing objective*. If the principle of contradiction fails to hold now, there will be cases in which A is and is not B at the same time. Consequently, under these conditions the proposition "A is B" would be false, if A were not B and also contained *no contradiction*. The principle of contradiction can in no way be derived from this definition of falsity.

19. Every proof of the principle of contradiction must take into account the fact that there are *contradictory* objects (e.g., the greatest prime number). In the *most general* formulation: "the same characteristic cannot belong and not belong to an object at the same time" *is in terms of the principle of contradiction most certainly false.*[14] It could only be true, and then it would also be proven *formally*, if the word "object" is to designate only objects which are *free from contradiction*. The question arises, how-

[14] So far as I know, Meinong first put this proposition forward. At the occasion of certain critical observations of B. Russell's, Meinong expressed himself in the following way (*Über die Stellung der Gegenstandstheorie im System der Wissenschaft*, [Leipzig, 1907], p. 16): "B. Russell lays the real emphasis on the fact that by recognizing such (*scil.* impossible) objects the principle of contradiction would lose its unlimited validity. Naturally I can in no way avoid this consequence. . . . Indeed the principle of contradiction is directed by no one at anything other than the real and the possible."

ever, whether such objects are available at all, especially whether the possible and the real contain no contradiction.

(a) *Constructive* abstractions [*Begriffsbildungen*] (existence-free objects according to Meinong), such as numbers, geometric figures, logical, and ontological concepts, etc.,—I call them "constructive" as opposed to "reconstructive" or empirical concepts which are supposed to represent reality—have often proven to be contradictory upon closer examination. One thinks, for example, of the squaring of the circle, of the trisection of an arbitrary angle, of the difficulties of transfinite set theory, etc. Hence, the possibility is by no means excluded that constructions which count today as free of contradiction nevertheless contain a deeply hidden contradiction which we have not yet been able to discover. And even if it should be just as certain as true that all constructions were "free creations of the human spirit" [15] and that it lies in our power to prescribe an existence-free object for any arbitrary characteristic, in spite of that we could not demonstrate absence of contradiction [*Widerspruchslosigkeit*] on their behalf. For, while we do "create" them, innumerable relations arise "by themselves" among them, which no longer depend on our will. A newly discovered contradiction by B. Russell,[16] which touches on the logical foundations of mathematics, demonstrates that we encounter completely unexpected and unexplained difficulties with such constructions.

(b) Actual objects and reconstructive abstractions, insofar as they correspond to reality, appear to be placed beyond contradiction. *In fact there is known to us no single case of a contradiction existing in reality.* Indeed it is generally impossible to suppose that we might meet a contradiction in *perception*; the negation which inheres in contradictions is not at all perceptible [*wahrnehmbar*]. Actually existing contradictions could only be

[15] The expression stems from Dedekind, *Was Sind und Sollen die Zahlen?* Forward.

[16] Cf. Russell, *The Principles of Mathematics*, vol. I (Cambridge, 1903), ch. X, and Frege, *Grundgesetzte der Arithmetik*, vol. II (Jena, 1903), Nachwort, p. 253. Further, K. Grelling and L. Nelson, *Bemerkungen zu den Paradoxen von Russell und Burali-Forti,* Abh. d. Fries'schen Schule, N.F., vol. II (1908).

inferred [*erschlossen*].—One might not forget, however, that from oldest times contradictions were suspected in the *continuous change* to which the entire world is ceaselessly subjected in constant becoming, arising, and passing away. Whether these suspicions can ever be confirmed seems to be improbable; one will always find ways and means eventually to dismiss inferred contradictions. *But one will never be able to assert with full definiteness that actual objects contain no contradictions.* Man did not create the world and he is not in a position to penetrate its secrets; indeed, he is not even lord and master of his own conceptual creations.

From (a) and (b) it is clear that a *real* [*realer*] proof of the principle of contradiction, i.e., a proof which would relate to an exact investigation of the actual and the possible *cannot be carried out.*

20. The principle of contradiction has, to be sure, no logical worth, since it is valid only as an assumption [*als Annahme*]; but as a consequence it acquires a *practical-ethical value*, which is all the more important. *The principle of contradiction is the sole weapon against error and falsehood.* Were we not to recognize this principle and hold joint assertion and denial to be possible, then we could not defend other propositions against false or deceitful propositions. One falsely accused of murder could find no means to prove his innocence before the court. At most, he could only manage to prove that he had committed no murder; this negative truth cannot, however, remove its contradictory positive from the world, if the principle of contradiction fails. If just one witness is found who (not shirking from committing perjury) implicates the accused, his false assertion can in no way be contradicted and the defendant is irretrievably lost.

From this one sees that the necessity of recognizing the principle of contradiction is a *sign of the intellectual and ethical incompleteness of man.* This fact, however, far more than anything else is in a position to call attention to and to justify our mistrust about the *logical* worth of this principle.

Even if not clearly recognized, it appears that even Aristotle at least sensed the practical-ethical worth of the principle of contradiction. At a time of the political decline of Greece, Aristotle

became the founder and investigator of systematic, scientific, cultural work. Perhaps he saw in that consolation for the future and the future greatness of his nation. For him, it must have been a prescription to hold high the value of scientific research. Denial of the principle of contradiction would have opened door and gate to every falsity and nipped the young, blossoming science in the bud. Hence, the Stagirite turns against the opponents of the principle with forceful language in which one can trace an internal fervor, against the eristic thinkers of Megara, the cynics of the school of Antisthenes, the disciples of Heraclitus, the partisans of Protagoras; and he battles with all of them for a theoretical principle as if for personal goods. He might well have himself felt the weaknesses of his argument, and so he announced his principle a final *axiom*, an unassailable *dogma*.